Division E

Mathematical Olympiads for Elementary and Middle Schools

DECEMBER 13, 2005

Contest 2

SOLUTIONS AND ANSWERS

2A *Items in parentheses are not required.*

45, 54
(both required)

2A METHOD 1: *Strategy: Consider the tens digits first.*
The numbers are between 40 and 60, so the tens digits are 4 or 5. Because the numbers are palimages, 5 and 4 are the only possible units digits. The numbers are different, so **the numbers are 45 and 54.**

FOLLOW–UP: *A number is called a* **PALINDROME** *if it reads the same when written forwards or backwards. (For example 12321 is a palindrome.) How many palindromes are there between 1000 and 2000?* [10].

2B

90
(times)

2B METHOD 1: *Strategy: Find the number of 8-second periods in the interval.*
One minute has 60 seconds, so two minutes have 120 seconds. Because 120 ÷ 8 = 15, a two-minute interval has 15 periods of 8 seconds each. Because the cricket chirps 6 times in each period, **the cricket chirps** 15 × 6 or **90 times in two minutes.**

METHOD 2: *Strategy: Use a proportion.*
Set up a proportion using chirps per second.
Let N = the number of chirps in 120 seconds.
Then $\frac{\text{chirps}}{\text{sec.}} = \frac{6}{8} = \frac{N}{120}$. Solve to get N = 90.
The cricket chirps 90 times in two minutes.

FOLLOW–UP: *Jim can cut a log into 5 pieces in 6 minutes. At that rate, how long will it take him to cut a log of the same thickness into 25 pieces?* [36 min; (How many cuts are made?)]

2C

(**X** *is*)
7

2C METHOD 1: *Strategy: Find the "magic sum."*
To find the sum of the numbers in each row ("the magic sum"), divide the sum of all the numbers by the number of rows. 1 + 3 + 5 + . . . + 15 + 17 = 81, so the magic sum is 81 ÷ 3, or 27.

Since 5 + A + 13 = 27, square A contains 9. Then 1 + 9 + B = 27, so square B contains 17. Finally, X + 17 + 3 = 27, so **7 goes in the square marked "X".**

	1	
5	A	13
X	B	3

METHOD 2: *Strategy: Find the number in the middle square.*
The mean of the numbers 1, 3, 5, 7, 9, 11, 13, 15 and 17 is 9. Therefore, 9 is the value in the middle box of this 3 × 3 magic square. The sum of the numbers in the middle row then is 27, which is the magic sum. Proceed as in Method 1 to find the square marked "X" contains 7.

2D

144
(cm)

2E

156
(pages)

FOLLOW-UPS *on back*

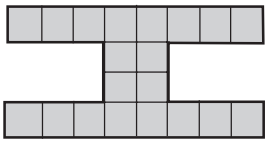
FOLLOW–UPS: *(1) Complete the magic square below using the first nine even numbers.*

12		
	6	

[The rows, from left to right, are 12, 14, 4; 2, 10, 18; 16, 6, 8] *(2)* ***EXPLORATION:*** *Note that the sum of the first nine odd numbers is 81. What is the sum of the first ten odd numbers?* [100] *The first eleven?* [121] *What is the sum of the odd numbers from 1 through 27?* [196]. *From 1 through 99?* [2500]

2D *Strategy: Determine the length of the side of a small square.*

The figure consists of 20 small congruent squares and the area of the entire figure is 320 sq cm, so the area of each small square is 16 sq cm. Then the side of each square is 4 cm. Measured in sides, the perimeter of the entire figure is 36. **The number of cm in the perimeter of the entire figure is** 36 × 4 or **144.**



2E *Strategy: Count the number of 7s in an organized way.*

METHOD 1: From pages 1 through 100, the digit seven appears 10 times as a units digit and 10 times as a tens digit (the 70's) for a total of 20 times. The remaining 5 sevens are used to number pages 107, 117, 127, 137, and 147. The next page that requires a seven and therefore the first page that cannot be numbered is page 157. Therefore, **the largest number of pages the book can have is 156.**

METHOD 2: This table separates the numbers in which 7 appears in the tens place from the rest of the numbers. Adjust the cumulative total once it passes 25 sevens.

pages	Number of Sevens			
	units place	tens place	subtotals	cum.
1 to 69	7	0	7	**7**
70 to 79	1	10	11	**18**
80 to 169	9	0	9	**27**

Numbering up to page 169 requires 27 sevens. This is two more than are available. The two pages less than 169 that cannot be numbered are pages 167 and 157. The largest number of pages the book can have is 156.

NOTE: *Other problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*