Division E

Mathematical Olympiads

NOVEMBER 15, 2005

for Elementary and Middle Schools

Contest 1

1A ***Time: 3 minutes***

What number can replace the square to make the statement true?

$$5 \times 11 = \square + 12$$

1B ***Time: 5 minutes***

Suppose N is a whole number.

For how many values of N is $\frac{60}{N}$ also a whole number?

1C ***Time: 6 minutes***

The sum of the three-digit number AAA and the two-digit number BB is the four-digit number CD6E. A, B, C, D, and E are different digits. What four-digit number does CD6E represent?

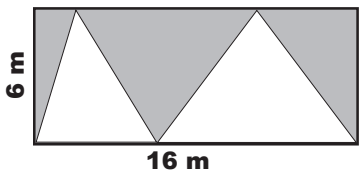
$$\begin{array}{r} A\ A\ A \\ +\quad B\ B \\ \hline C\ D\ 6\ E \end{array}$$

1D ***Time: 5 minutes***

A bowl contains 100 pieces of colored candy: 48 green, 30 red, 12 yellow, and 10 blue. They are all wrapped in foil, so you do not know the color of any piece of candy. What is the least number of pieces you must take to be sure that you have at least 15 pieces of the same color?

1E ***Time: 7 minutes***

What is the area, in square meters, of the shaded part of the rectangle?



Please fold over on line. Write answers on back.

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Contest **1**

1 A *Student Name and Answer*

1 B *Student Name and Answer*

values of *N*

1C *Student Name and Answer*

1 D *Student Name and Answer*

pieces

1 E *Student Name and Answer*

sq m

Please fold over on line. Write answers in these boxes.

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Contest 1

SOLUTIONS AND ANSWERS

Items in parentheses are not required.

1A 43

1A *Strategy* *Evaluate the left side of the equation.*

5 × 11 = 55, so 55 = ☐ + 12.

Then ☐ = 55 – 12 = 43.

To make the statement true, replace the square by 43.

***FOLLOW-UP**: Given 15 × ☐ = ☐ + 84.*
What one number can replace <u>both</u> squares to make the statement true? [6]

1B 12 *(values of N)*

1B *<u>Strategy</u>: Make an organized table of factor pairs.*

For $\frac{60}{N}$ to be a whole number, *N* must be a factor of 60.

The factor pairs for 60 are listed in the table:

Factor pairs
1 × 60
2 × 30
3 × 20
4 × 15
5 × 12
6 × 10

$\frac{60}{N}$ is a whole number for 12 values of N.

***FOLLOW-UPS:** (1) How many factors (divisors) does each of the following have: 120; 180; 300? (Note that these are 60 × 2, 60 × 3,* and *60 × 5.)* [16 factors; 18 factors; 18 factors] *(2) (EXPLORATION) Find the 3 least numbers that have exactly three factors. What property do they share?* [4, 9, 25; They are squares of prime numbers.] *Repeat for numbers that have exactly five factors.* [16, 81, 625; they are fourth powers of primes.]

1C 1065

1C *<u>Strategy</u>: Work left to right.*

The sum of a three-digit number and a two-digit number is less than 1100. The sum is greater than 1000 only if the three-digit number is greater than 900. So C = 1, A = 9 and D = 0. The problem now is

$$\begin{array}{r} 9\,9\,9 \\ +\ B\,B \\ \hline 1\,0\,6\,E \end{array}$$

The addition in the tens column, 9 + B, yields a units digit of 6. This can happen two ways: either B is 7 and there is no "carry" from the ones column, or B is 6 and there is a "carry". Whether B is 7 or 6, there is a carry from the ones column. So B is 6, and E is 5, making the addition 999 + 66 = 1065. **The four-digit number is 1065.**

1D 51 *(pieces)*

1E 48 *(sq m)*

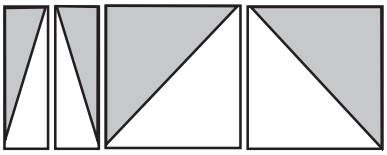
1D *Strategy: Consider the worst case.*

Determine the largest number of pieces you could take and still not have 15 of the same color. You could take all 12 yellow, all 10 blue, 14 of the green, and 14 of the red pieces and still not have 15 of the same color, for a total of 50 pieces. The next piece you take, whether green or red, gives you 15 matching pieces. **The least number of pieces you must take to be sure that you have 15 pieces of the same color is 51.**

Follow-Up: *Ana has 8 pennies, 3 quarters, 6 nickels, and 5 dimes in her piggy bank. She needs a dollar to buy a card. Ana shakes the bank so that the coins come out one at a time. What is the greatest number of coins Ana can shake out and still not have enough for the card?* [19] *What is the fewest number of coins that will get her the card?* [6]

1E *Strategy: Split the region into simpler figures.*

Draw line segments as shown to split the given rectangle into four smaller rectangles.



Each small rectangle is cut in half by its diagonal. Half of each small rectangle is shaded. Therefore half of the original rectangle is shaded. The area of the original rectangle is 16 × 6 or 96 sq m, so **the area of the shaded part of the rectangle is 48 sq m.**

NOTE: *Other problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*